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Some remarks on scatter plots generation procedures for facility layout

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Scatter plots of facilities on the plane are derived from a heuristic procedure proposed by Drezner. Features of this approach have been investigated. It is shown that scatter plots generated by the 'eigenvectors method' could differ in quality. It is shown that this quality depends on the input data structure. By the comparison of eigenvector results with those obtained by a special algorithm proposed in this work, general conditions are determined for rational applications of the Drezner approach.

1. Introduction

In papers by Drezner (1980, 1987), procedures were proposed for the generation of reasonable facility layouts in the form of scatter plots. In the analytical approach of Drezner (1980), scattered plots are a result of the first phase only of the algorithm but, in practice, as noted in his second paper (Drezner 1987), they proved to be very useful, e.g. in the work of architects. Drawings of facilities scattered on a plane may be a useful benchmark for them in urban planning, locating industrial plants, etc. The starting point for construction of scattered layouts is the formulation of the objective function as:

$$\min\left\{ f = \sum c_{ij} d_{ij} \Big/ \sum d_{ij} \right\}, \tag{1}$$

where $c_{ij} \geq 0$ denotes the linkage between facilities ij, while d_{ij} denotes the distance between them. The heuristic proposed by Drezner (1987) is very effective and is based on the properties of eigenvectors and the eigenvalues of matrices. Namely, if in problem (1) d_{ij} is substituted by d_{ij}^2, which Drezner considers to be 'intuitively reasonable', we have:

$$\min\left\{ ff = \sum c_{ij} d_{i,j}^2 \Big/ \sum d_{ij}^2 \right\}. \tag{2}$$

Problem (2) has its optimal solution as a straight line. The coordinates of the solution (the same for x and y) are the successive elements of the eigenvector connected with the second lowest eigenvalue of the matrix S in which $s_{ij} = -c_{ij}$ for $i = j$ and $s_{ii} = \sum_j c_{ij}$ for all i. A good solution on a plane may be obtained as the coordinates, y, of the eigenvector elements, connected with the third lowest eigenvalue of matrix S, since these are the coordinates of the best solution, which is orthogonal to the x coordinates vector. In accordance with this idea, the generating algorithm is extremely simple. Having the set of links c_{ij}, it is enough to:

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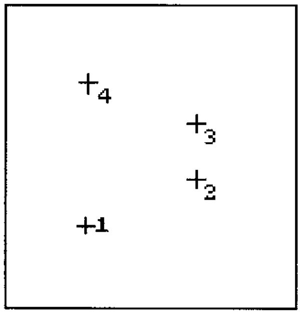

Figure 1. Scatter plot of 'corridor problem' from table 1 obtained by Drezner's approach.

(a) construct the matrix S;
(b) calculate the eigenvalues and eigenvectors;
(c) select two vectors connected with the second and third lowest eigenvalues, treating them as the coordinates x and y of the solution on a plane.

Since Drezner verified the scattered solutions only by mapping them into the constrained regular patterns and by comparing them with the ones obtained in the well known 'classic' method (on the same constrained, regular patterns), it is appropriate to investigate how effective is the proposed 'pure' heuristic. Particularly interesting is the relation of the objective function f (problem 1) and ff (problem 2) since the proposed treatment of changing d into d^2 is controversial and, in some sense, the opposite of the well known linearization methods of operational research.

A motivation for this research was a chance assessment of an application of Drezner's algorithm to the solution of a 'corridor design problem'. The problem was formulated as seeking the optimum shape (on a plane) of a corridor (or even a building) where material flow (of the intensity $c_{ij} = 1$) occurs only between successive rooms located along a corridor, i.e. from 1 to 2, from 2 to 3, etc. The main conclusion drawn in that work was that the corridors (and, consequently, the buildings) should be optimally built in a U-shape, yet it is unnecessary to know the details of scattered plots theory to infer that the conclusion is senseless. However, direct application of the algorithm described above generates such a solution. An example for the data given in first column of table 1 is presented in figure 1, in which 'U' is rotated 90° degrees left. It is evident that, in accordance with general layout theory (i.e. minimization of the function $\sum c_{ij} d_{ij}$), it should be precisely indifferent what shape the building will have for this 'corridor problem' defined in table 1, as the operating costs of the system of figure 1 depend only on the distances of individual points.

This is not the case if a criterion is the function (1). A U-shape solution here is, in general, 'not good', as simply 'bending' the arm of the U outwards increases the denominator of the f function (the numerator being unchanged) and, thus, the objective function value is decreased. The formulation of the problem in using objective function (1) is, therefore doubtful, and even more doubtful is the solution obtained with eigenvectors (you could imagine towns of the optimum U-shape with buildings of such shapes).

The example presented is admittedly only loosely connected with the above-mentioned controversial treatments of substitution of the distance by its square,

Facility	Corridor shape	Dreznerl	Drezner2	Drezner3	10–80	15-80	20–80	25–90	30–80
	Lists of interconnected facilities in basic described experiments								
1	2	8, 10, 15	5, 6, 10, 12, 17, 18	7, 8, 9	3, 4, 5	3, 15	2, 8, 9, 13, 20	3, 5, 7	2, 9, 13, 17
2	1, 3	6, 14, 15	6, 7, 12, 13, 14, 16	4, 6, 8	3, 4, 6, 10	4, 8, 9, 12	4, 5, 9, 10, 12	4, 7, 24	4, 5, 9, 11
3	2, 4	11, 18, 19	8, 9, 15, 16	10, 13, 16	4, 5, 7	7, 11, 12, 1	4, 8, 9, 15, 16	9, 10, 12	4, 8, 9, 15, 20
4	3	6, 10, 13	9, 11, 15, 17	2, 15, 17	5, 6, 8, 9	5, 6, 9, 11	6, 9, 10	6, 8, 13, 14	6, 9, 10, 21, 25
5	—	10, 16, 18	1, 11, 17, 18	6, 18, 19	6, 7, 9	7, 9, 10	6, 7	—	6, 8, 11
6	—	2, 4, 14, 17	1, 2, 10, 12, 13, 19	2, 5, 11	7, 8	8, 12, 13	7, 19, 20	7, 23, 24	7, 19, 29
7	—	9, 13, 19	2, 13, 14	1, 12, 14	8	12, 14, 15	19	9, 12, 15, 18	19, 20
8	—	1, 15, 16	3, 14, 16	1, 2, 18		9, 10, 14	9, 13, 14, 15	11, 15, 17	9, 13, 14, 17
9	—	7, 12, 17	3, 4, 12, 15, 16, 17	1, 10, 11		—	—	10, 19	—
10	—	1, 4, 5, 16	1, 6, 18, 19	3, 9, 12		—	15, 17	21, 22	15, 21, 26, 30
11	—	3, 13, 18	4, 5, 17	6, 9, 19		14, 15	16, 20	14, 16, 18, 19	16, 20, 22, 26
12	—	9, 14, 17	1, 2, 6, 9, 16, 17	7, 10, 16		13, 14	13, 15, 17	14, 17	17, 21, 30
13	—	4, 7, 11, 19	2, 6, 7, 19	3, 15, 17		15	14, 18, 20	14, 15, 22	14, 18, 25, 28
14	—	2, 6, 12	2, 7, 8, 16	7, 17, 18, 19			15, 17, 18	17, 19, 22	15, 18, 29
15	—	1, 2, 8	3, 4, 9	4, 13, 16			20	16, 24, 25	20, 26, 30
16	—	5, 8, 10	2, 3, 8, 9, 12, 14	3, 12, 15			—	18, 19	—
17	—	6, 9, 12	1, 4, 5, 9, 11, 12	4, 13, 14			—	20, 22, 23	22, 23, 26
18	—	3, 5, 11	1, 5, 10	5, 8, 14			—	19	24, 25, 26
19	—	3, 7, 13	6, 10, 13	5, 11, 14			20	21, 24	30
20								22, 25	25, 29
21								24	26
22								25	27, 29
23								24	27, 28
24									29, 30
Results obtained by Drezner's algorithm									
	f	5, 97	11, 21	7, 49	23, 14	16, 7	9, 13	8, 37	6, 05
	ff	2, 14	4, 49	3, 61	13, 3	9, 12	4, 1	3, 54	2, 58

Table 1. The corridor problem and Drezner's three examples.

but it suggests the necessity of accurate analysis and major care in the application of the approach under discussion.

To assess Drezner's proposition more accurately, a simple algorithm was constructed, in order to obtain a reasonable solution of variable parameters and objective function values, since Drezner's algorithm gives only one solution for any given link matrix S.

2. 'Virtual force' algorithm

The solution generator operating principles were based on the ideas behind DISCON, as described in the paper by Drezner (1980). This approach was adapted to maximize flexibility of the new algorithm and the interactive opportunities of its computer implementation. The analytical approach was abandoned and replaced by the heuristic simulation of physical analogy. In the DIS phase of the DISCON algorithm, elastic disks interconnected with springs located in the system centre 'exploded' due to the elasticity forces outside and next, in the concentration phase, were pulled by the springs connected to the centre, producing a cluster wherein they stayed close to one another, strongly interconnected. The analogy proposed here is somewhat simpler but a little more abstract. Each facility is represented by a material point (of no dimension). In order to scatter the facilities randomly on a plane, all facilities could be initially located at the area centre at the beginning, and next, for each facility, a direction and distance on which each facility is to be moved would be chosen randomly. Such scattering corresponds to a possible (random) layout of non-interconnected facilities on a plane. Physically, it can be obtained in such a way that on each point there acts a pushing out force acting from the centre along the chosen direction. The force decreases when moving away from the centre and, at a point in distance of a pre-selected value, decreases to zero. This force may be called a 'scattering' one. The notion 'virtual' is used because of the difficulties in finding a real, physical realization of the complete idea. In particular, such difficulties can be imagined if it is assumed now that, along the path taken by each facility to the chosen location after a given period, there acts on the facility an attractive force proportional to the interconnections c_{ij} from each facility (interconnected with the latter). The forces may be different for each interconnected pair. In addition, each facility may have a defined radius from which it ceases to be attracted. The force between the facilities acts inverse proportionally to the scattering force (the farther the interconnected facilities, the stronger they are attracted). Facility travel from the centre to the chosen locations may be simulated on a computer by dividing the total distance into stages where each facility will be moved only a 'small step' in the direction of the resultant of the virtual forces from other facilities and the scattering force (from the centre of gravity of the complete system). This idea is realized by the algorithm pseudocode given below.

1 INPUT DATA

n—number of facilities;
Define facilities links matrix $L = [l_{ij}]$;
Define the layout area (0,0,$x$max, $y$max);
$C = (xc, yc)$—layout area centre coordinates ($xc = x\text{max}/2$; $yc = y\text{max}/2$);
Define variables

- 'buffer zone' size $r(i)$; (radii of zones where no virtual force from other facilities acts as a percentage of a layout area diagonal size);
- coefficient ε—the value proportional to which each facility will be moved from the layout area centre before forces from other facilities will be 'turned on'.
- coefficient β—the value proportional to which each facility will be moved in reaction to the virtual force in one simulation step. Physically it shows the distance (in the layout area units) of the facility movement when the virtual force value from other facilities is equal to 1. (The reader familiar with the computer simulation will notice that β could be interpreted simply as the duration time of a separate simulation step.)
- The above (ε and β) factors play a key role in the time of the solution completion and stability. In this simple algorithm version they must be determined in some experimental trials.

2 CHOOSING STARTING CONDITIONS

For $i = 1$ **to** n **do**
  begin
    choose randomly $x_i \in [0, x\max]$, $y_i \in [0, y\max]$;
    store $P(i) = [x_i, y_i]$;
    calculate $D(i) = \mathrm{SQR}((xc - x_i)^2 + (yc - y_i)^2)$;
    move facility i from point C in direction $P(i)$ by $\varepsilon\, D(i)$;
  end;

2 LAYING OUT

Repeat
  Calculate centre of gravity $W = (xw, yw)$ of the system and move each facility of
  $D = C - W$ so that the system has its gravity centre at C;
  For $i = 1$ to n **do**
    begin
      determine the resultant vector of 'virtual force' VF acting from other facilities, assuming that the force from each pair is directly proportional to the interconnection and distance of the facilities and acts only up to the distance equal to the 'buffer zone' sum;
      calculate distance $D(ci)$ between $P(i)$ and C. If the distance is lower than $D(i)$ (randomly chosen in the first step) than add the VFC vector of the force acting in the direction from C to $P(i)$, in a way proportional to $1/D(ci)$, to the virtual force resultant vector (VF = VF + VFC);
      move facility i in the direction of the resultant vector of value $\beta\,|\mathrm{VF}|$;
  end;
  calculate value of function f;
  calculate value of function ff;

Until the system is stabilized or the preset iteration number is attained.

Concatenation of virtual force vectors occurs in the normal way, i.e. such as for the physical force case; the vector lengths only are calculated in a specific way, needing some comment. To make the operation of the algorithm independent of the geometric size (scale) of the layout area, a rule was adapted so that the virtual force action zone for each facility pair i, j is such that, in a distance that is very high compared with the size of the buffer zone sum, the force is maximal (tends to 1), whereas, within the zones it is equal to 0, and this relation is maintained in each layout plan scale. Such assumptions are realized by the following formula for virtual force vector length calculation outside the buffer zones:

$$|\mathrm{VF}(ij)| = 1 - (r(i) + r(j))/D(ij), \tag{3}$$

where $D(ij)$ denotes the ij facility distance and $r(i)$ denotes the ith facility buffer zone radius.

One can notice that (3) describes a kind of a simple ‘normalized spring’ attracting a given facility pair.

In the case of the scattering force, the similar formula acts, on the basis of the assumption, so that a facility is moved from the system outwards with the maximum force near the centre and the minimum one near the chosen distance value. An appropriate formula is, here, as follows:

$$|\mathrm{VFC}(i)| = 1 - D(ci)/D(i). \tag{4}$$

Of course the relationship works only for $D(ci) < = D(i)$. An implementation of the idea made in Pascal shows additionally, graphically, the course of the objective function changes for f and ff. Display hard-copies are presented in figures 2, 3 and 4. The facilities are denoted with a cross symbol and numbers.

3. Experiments with Drezner’s examples

The first step of the research work performed was based on examples from Drezner’s paper. Table 1 shows the interconnections for three examples described as Drezner1, 2 and 3. It was assumed that each link had force 1. For each example, the solution was found first with the algorithm based upon eigenvectors. The solutions are given in graphic form in the paper by Drezner (1987) but the values of the objective function (1) are not given. It can be easily noticed that the presented virtual force procedure is very flexible since many parameters may by changed within it. In a series of pilot experiments it became clear that, from the viewpoint of the drawn target (validation of f and ff function relations), the greatest influence upon the variability in the generated solution is due to the relation of the buffer zone size ($r(i)$) with respect to the layout area size. Therefore, a percentage share index of buffer zone sizes for two facilities up to the length of the layout zone area diagonal (R) was introduced into the algorithm. It was assumed, then that each facility has the same buffer zone. For all experiments, the coefficients ε and β were determined empirically so that stable solutions are obtained in a relatively short time. Although this was not too precise a requirement, it meant that the f function stable state was obtained after not more then 100 steps. Both coefficients were preset at the same level equal to 10. It should be emphasized, however, that these are not universal values since they express unit movement in a specific layout area scale.

Since, even in the case of determination of all input variables, the algorithm produces a different solution due to the choice made in the first step, ten experiments

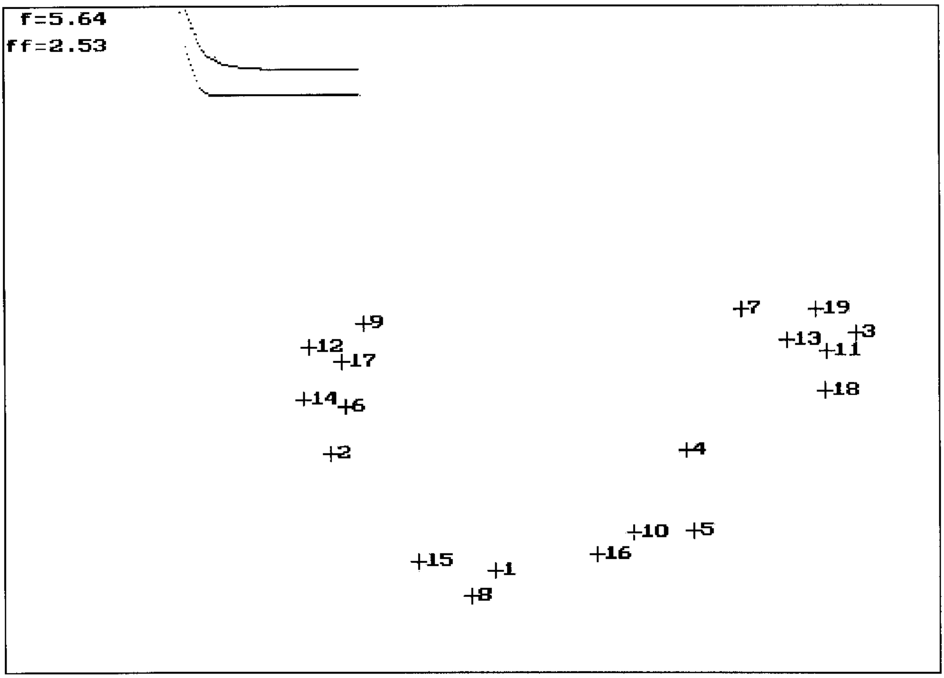

Figure 2. The screen copy of the 'Virtual Force' algorithm implementation. Exemplary result for the Drezner1 case (R= 5%). See the f and ff functions' dynamic plots in the upper left part of the screen.

Drezner1	R	3	5	8	10	13	15
	f (mean)	4, 90	5, 61	6, 14	6, 55	6, 81	6, 98
	ff (mean)	3, 06	2, 71	2, 44	2, 54	2, 55	2, 59
	opt f (Drezner's algorithm)			5.97			
	opt ff (Drezner's algorithm)			2.14			
Drezner2	R	3	5	8	10	13	15
	f (mean)	11, 13,	11, 18	11, 23	11, 29	11, 41	11, 52
	ff (mean)	4, 67	4, 63	4, 62	4, 63	4, 69	4, 76
	opt f (Drezner's algorithm)			11.21			
	opt ff (Drezner's algorithm)			4.49			
Drezner3	R	3	5	8	10	13	15
	f (mean)	5, 45	6, 34	6, 94	7, 83	8, 13	8, 44
	ff (mean)	3, 77	3, 54	3, 33	3, 49	3, 61	3, 81
	opt f (Drezner's algorithm)			7.49			
	opt ff (Drezner's algorithm)			3.61			

Table 2. Mean results of experiments with Drezner's examples and the 'Virtual Force' algorithm.

were run for six selected values of the index R for each of the three Drezner's examples.

The mean results of studies are listed in table 2. An example plot for the 'Drezner1' case is depicted in figure 5. In each case qualitatively similar relations

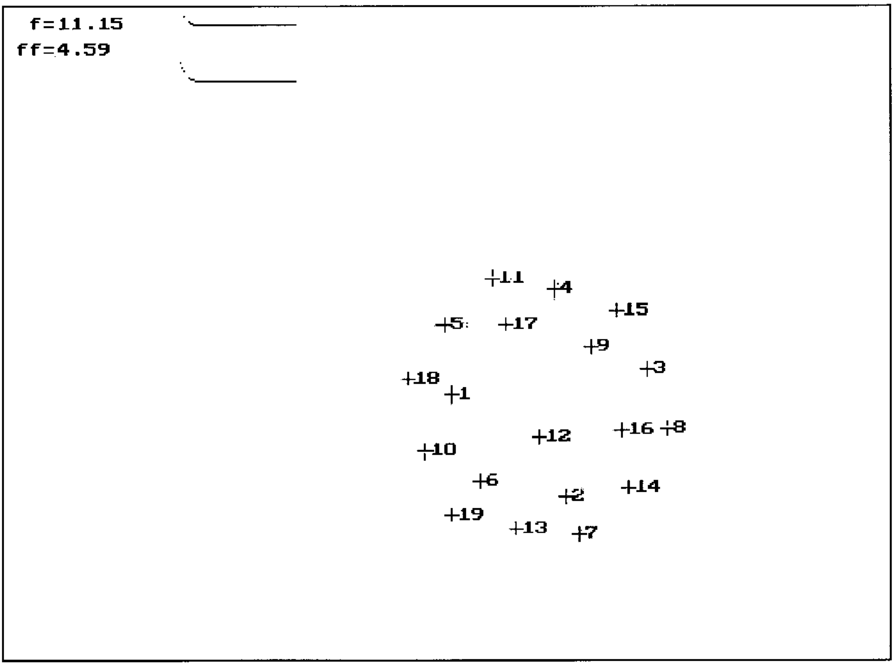

Figure 3. The screen copy for sample results of the Drezner2 case ($R = 5\%$).

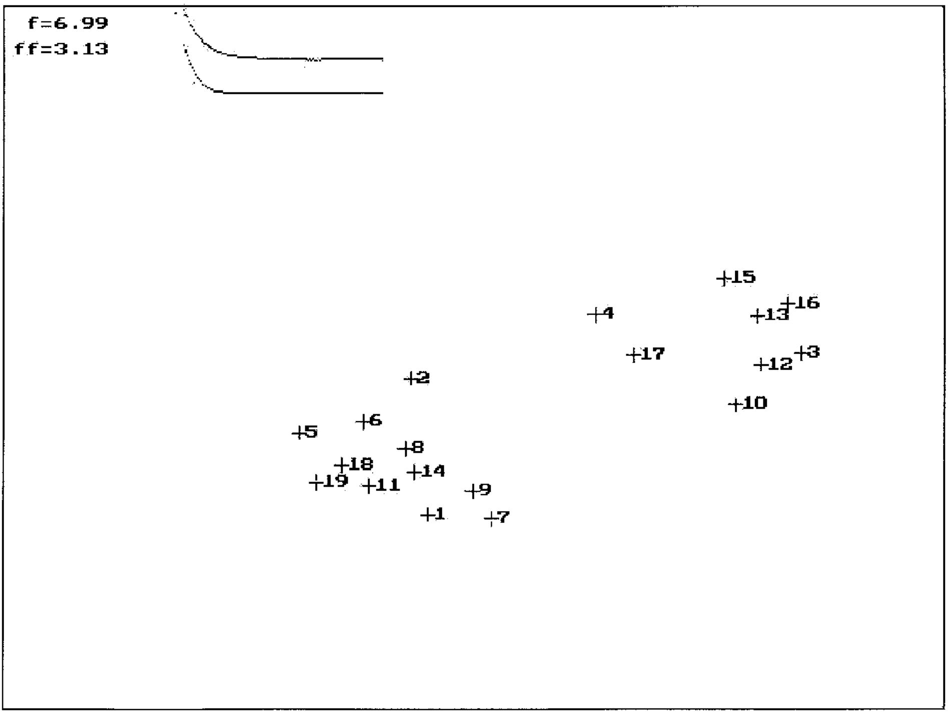

Figure 4. Sample results of the Drezner3 case ($R = 8\%$).

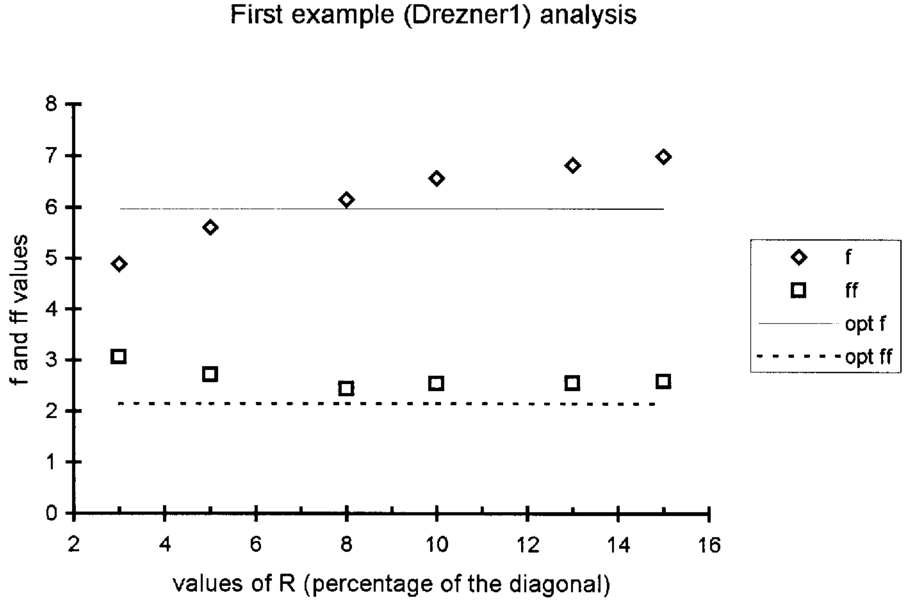

Figure 5. Relationship of f and ff versus buffer zone dimension (R) for the Drezner1 data. Opt f and opt ff represent the respective Drezner's algorithm results.

were obtained. First, in each case the presented algorithm quite easily obtains better solutions under criterion (1) than the eigenvector method. Examples of such solutions are presented in figures 2, 3 and 4. These better solutions occur for relatively small buffer zones (the area diagonal in the experiments was kept constant). However, the most interesting result is the relationship between f and ff, *as* shown in the plots in figure 6. It appears that the heuristic proposed here operates well only for smaller R values but in the neighbourhood of the best solutions (in the sense of f), the two criteria act in opposition! The above seems to be a basic theoretical weakness of Drezner's approach since it is directed on minimization of the ff objective function. On the other hand it seems also to be a main reason for the 'virtual force' algorithm observed efficiency.

It should be also noted that while the relationship between f and ff is very clear in the Drezner1 and Drezner3 examples (table 1), it is considerably weaker in the Drezner2 example (the ff scale is elongated in the plot of figure 6(b)). Although this example, as Drezner notes, is artificially devised, the basic difference with respect to the other two is the stronger and very uniform interconnections of facilities (column Drezner2 in table 1). In this example, the ff functions change minimally with changing index R. The changes are statistically insignificant on the level $p = 0.05$ in contrast with the very significant relations in the Drezner1 and Drezner3 examples ($p < 0.001$). In this case, the solutions with the Virtual Force algorithm (VF) and eigenvector methods are visually very similar.

This observation suggested a series of pilot tests, whereupon the problem based on the Drezner1 example was modified by randomly adding new connections (originally the system has 30 links—see table 1). In figure 7 the ff-f relation is listed for successive links numbers (N). The relations were obtained for mean values from the five best results in a series of ten trials for each example and it can be seen how the

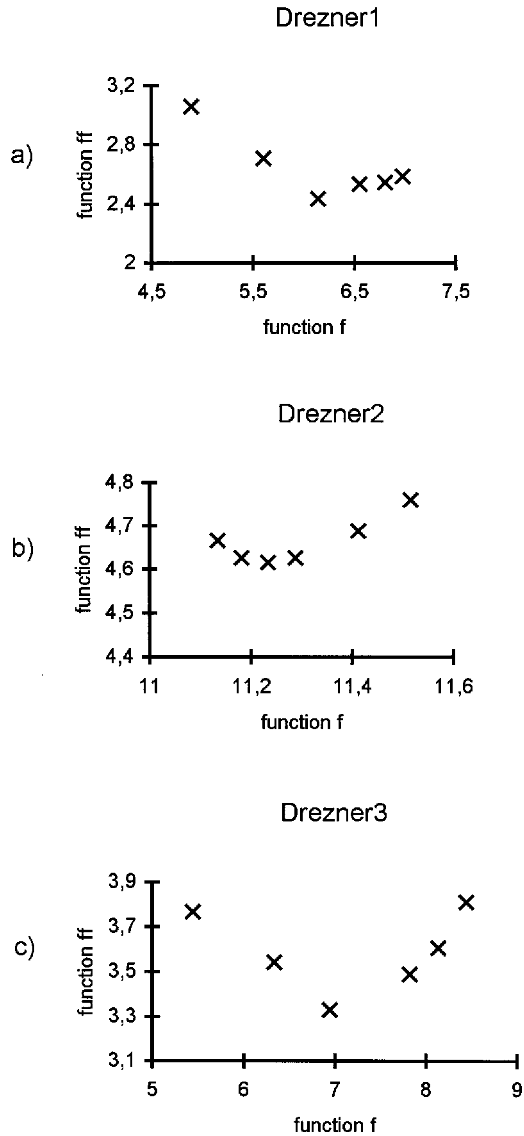

Figure 6. Relationship between f and ff in the experiments for the Drezner1 Drezner3 data (*a-c*).

apparent inconsistency in the f and ff criteria assessments fails when the connection number increases.

An interesting question arising in this situation involves the limit above which the inconsistency of assessments of both criteria does not occur, since this is essentially a question of the validity of Drezner's heuristics (in the step of exchanging f and ff). In the tested examples it appeared that this limit is close to the number expressing the theoretical maximum number of neighbours allowed for a given set of facilities being laid out, given by Moore (1980). This number, implied by Euler's equation, defines the maximum number of edges (E) that may be realized in a form of a planar graph at a given number of vertices (V):

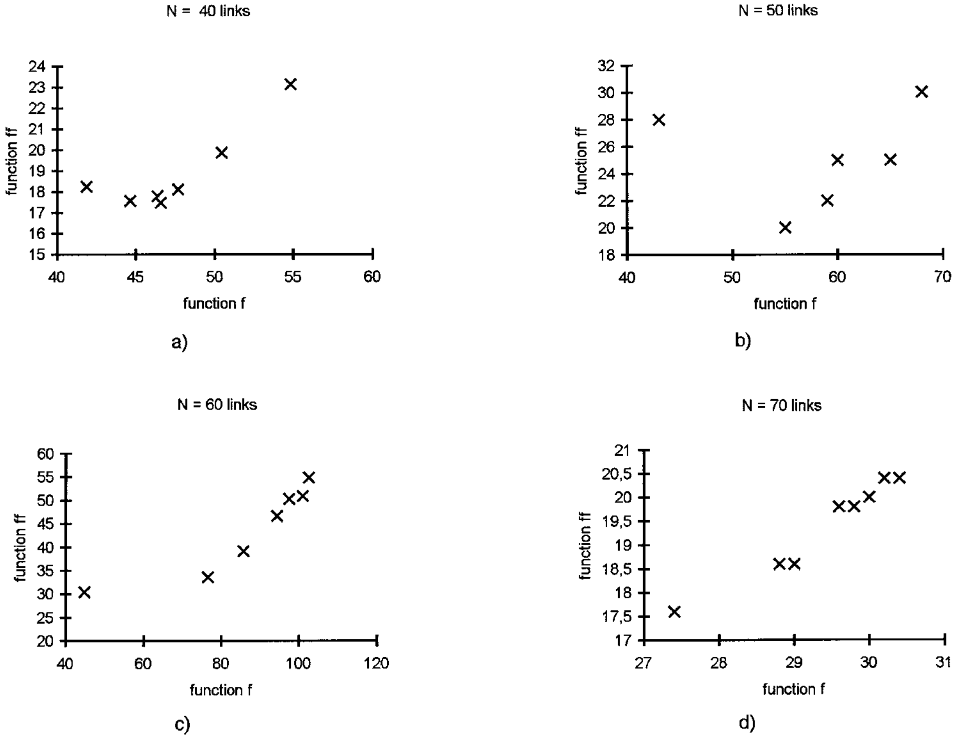

Figure 7. Results of the experiments based on the Drezner1 data in which ten new links were added in case (a), 20 links in case (b), 30 in case (c) and 40 in case (d).

$$E_{\max} = 3V - 6 \tag{5}$$

since the planar graph models the adjacency of facilities on a plane. For the examples investigated here, $E_{\max} = 3 \times 19 - 6 = 51$. From figure 7($a$)–($d$) it can be seen that, above this limit, both functions assess various solutions consistently. Obviously, this empirical observation remains as a conjecture needing further validation.

4. Experiments with randomly generated problems

To find some more characteristics of the observed relationships the second set of experiments was undertaken. The main goal of these experiments was to find if the above relationships are independent of the number of the facilities being laid out and also of the structure of links. As it was pointed out in Drezner's work, two of the studied examples were designed in a special way—knowing the optimal layout. Here, a new set of interrelationships was created 'almost randomly'. The only restriction was to avoid unconnected facilities in the S matrix because such a situation leads to artificially good criteria values as a result of moving the unconnected element far from the group (see equations (1) and (2)). First, three different links matrices S were created with 10, 20 and 30 elements respectively. These basic matrices have 80% of $E_{\max}$ interconnections defined. They are shown in table 1 as '10–80', ' 20–80' and '30–80' columns respectively. Adding (randomly) new links, a set of respective $E_{\max}$ problems was created, subtracting the appropriate link amounts to 60% of the $E_{\max}$ problems being defined.

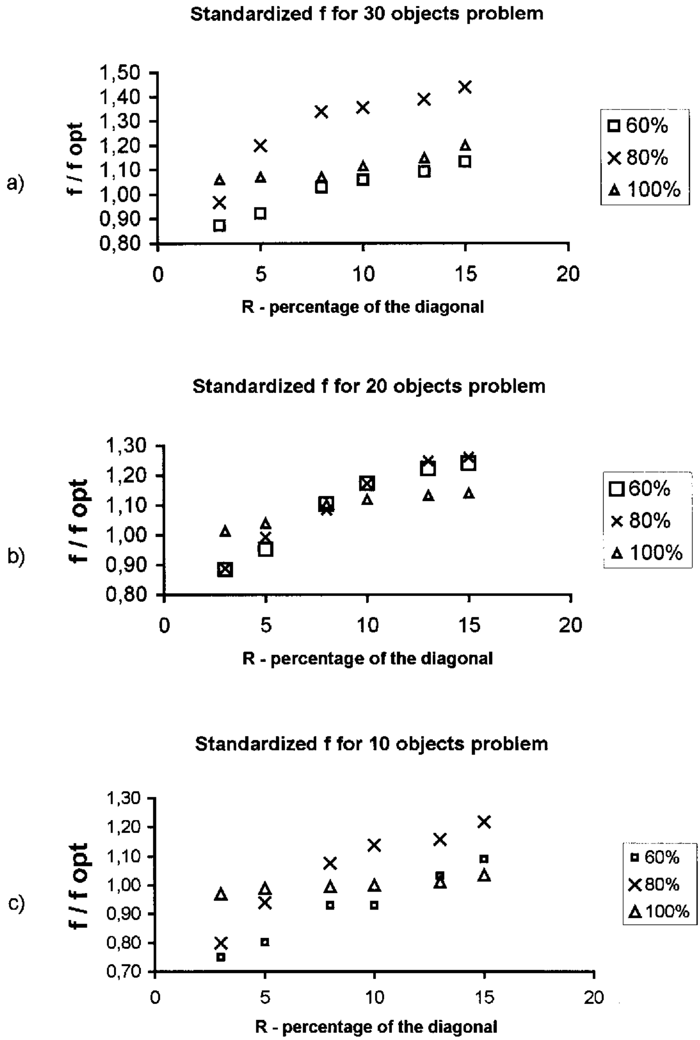

Figure 8. Standardized best results (f/f-opt) obtained by the VF algorithm for nine randomly generated problems versus buffer zones (R) and E_{max} percentage.

All nine problems were first solved using the eigenvector method. After that, the Virtual Force algorithm was applied ten times for each of six levels of the R value (3–15% of the layout area diagonal) in each of the problems defined. Best values of f and the appropriate ff were then chosen and standardized by dividing them by f-opt and ff-opt (results of the eigenvectors' method) respectively. The relationship between these standardized f and ff was studied carefully for each problem. It was found that it is similar to that derived in the first experimental series (figure 7) although in the 30-element problem the inconsistency between f and ff exists even for a 100% E_{max} case.

Figure 8 shows standardized best results (f/f-opt values) versus levels of R (buffer zones) for all problems with respect to the E_{max} level. These are also established in table 3. One can notice than a value lower then 1 in these figures means that f

Percentage of R	30 object problem†			20 object problem†			10 object problem†		
	60%	80%	100%	60%	80%	100%	60%	80%	100%
3	0, 87	0, 97	1, 06	0, 88	0, 89	1, 01	0, 75	0, 80	0, 97
5	0, 92	1, 20	1, 07	0, 95	0, 99	1, 04	0, 80	0, 94	0, 99
8	1, 03	1, 34	1, 07	1, 10	1, 08	1, 10	0, 93	1, 08	0, 99
10	1, 06	1, 36	1, 12	1, 17	1, 17	1, 12	0, 93	1, 14	1, 00
13	1, 09	1, 39	1, 15	1, 22	1, 25	1, 13	1, 03	1, 16	1, 01
15	1, 13	1, 44	1, 20	1, 24	1, 26	1, 14	1, 09	1, 22	1, 04
f opt	3, 57	6, 05	10, 18	5, 97	9, 13	15, 19	15, 6	23, 14	36, 01

†% of Emax.

Table 3. Best f objective function results for each R level obtained by the 'Virtual Force' algorithm with randomly generated problems.

(obtained by the VF algorithm) was better than f-opt (obtained by eigenvectors). This is also in accordance with the previous results.

The relationship that is the general conclusion from the above research is that the level of standardized f depends (almost proportionally) on the level of R when applying the VF algorithm.

It is clear from figure 8 that the level of standardized f depends in some way also on the level of E and the number of facilities being laid out. The 60% level of E_{max} seems to be the best for the efficiency of the proposed VF algorithm—it gives the best results for 3% R and 60% E_{max} for all cases. Relationships between 80 and 100% levels are not so clear. Perhaps also other parameters play a role for the obtained results.

To clarify and verify the above observations statistically, some additional experiments were designed to determine: (a) whether the number of elements influences Drezner's method results, (b) which way the number of links (measured as a percentage of E_{max}) influences results and (c) whether the results depend on the links structure.

To answer the above questions, additional problems of 15 and 25 elements were defined in a similar way to the method described above (table 1). Two different structures were proposed ('almost randomly' as before) for each problem of the same size and the same number of links (percentage of E_{max}). The input S matrix for each case was characterized by its diagonal elements variance and range (max s_{ii} min s_{ii}). The best results of the VF algorithm in the form of standardized f were established together with those obtained before for 10, 20 and 30 element problems. Matrix S characteristics were also computed for the previous problems. All results are put together in table 4. Statistical analysis was used to determine the basic dependencies between problem characteristics and obtained results. Analysis of variance has shown that the number of links (% of E_{max}) has the most significant influence on the results ($p < 0.001$) while the number of elements is less important ($p = 0.057$). However, there is no significant difference (according to the Student t-test) between two structures of the same size and linked pairs of the last series number, but they were generated in a random way and so structure parameters were not controlled (excluding the number of links of course).

Figure 9 shows plots of the main relationships for the obtained results. The two analysed factors (problem size and E_{max} percentage) do not explain the results com-

Problem size	% of Emax	Variance	S-matrix diagonal range	Best (minimal) f/f opt	Index of regularity
10	60	1, 11	3	0, 74	0, 7
	80	3, 33	6	0, 8	0, 5
	100	0, 84	3	0, 97	0, 8
20	60	0, 78	2	0, 72	0, 67
	60b	1, 78	4	0, 79	0, 67
	80	1, 12	4	0, 76	0, 67
	80b	2, 98	6	1, 10	0, 73
	100	1, 31	4	1, 02	0, 87
	100b	0, 6	2	1, 04	0, 67
20	60	1, 85	4	0, 88	0, 4
	80	1, 92	4	0, 89	0, 35
	100	2, 69	5	1, 01	0, 6
20	60	1, 17	4	0, 89	0, 4
	60b	1, 37	5	0, 99	0, 44
	80	2, 3	6	0, 82	0, 32
	80b	1, 14	4	0, 97	0, 6
	100	3, 21	7	1, 17	0, 28
	100b	4, 67	9	1, 12	0, 52
30	60	1, 67	5	0, 87	0, 67
	80	2, 1	6	0, 97	0, 33
	100	3, 76	5	1, 03	0, 47

Table 4. Best results and parameters analysed in the second series of experiments with randomly generated problems.

pletely. The correlation analysis of the matrix *S* diagonal elements variance and ranges with experiments results has shown that there is the most significant relationship between ranges of the *S* matrix diagonal elements and the standardized *f* ($r = 0.5$). Figure 10(*a*) shows this relationship. The regression line is also plotted. R^2 shows that more then 25% of the changes in *f* is explained by ranges of the *S* matrix diagonal elements. Combining the two main results one can say that, generally, Drezner's approach gives better results (it is harder to improve them using the VF algorithm, and probably other algorithms) for problems which are large (taking into account the number of facilities) and have many interconnections between facilities that are not uniformly distributed.

5. Scattered plots regularity

The simple visual analysis of some results of the eigenvector approach in the form of scattered plots leads to the conclusion that facilities can be distributed more or less regularly in a layout region. Because the layout regularity in some practical cases could be even the objective, some analysis of this problem was performed in the last series of experiments.

A simple regularity index was first constructed and implemented on the computer.

Each plot was first closed in a minimal rectangular window with sides parallel to the *X*–*Y* axes. This window was divided into equally sized cells. The number of cells was, for each case, defined as a minimal number *K* such that *K* is greater than or

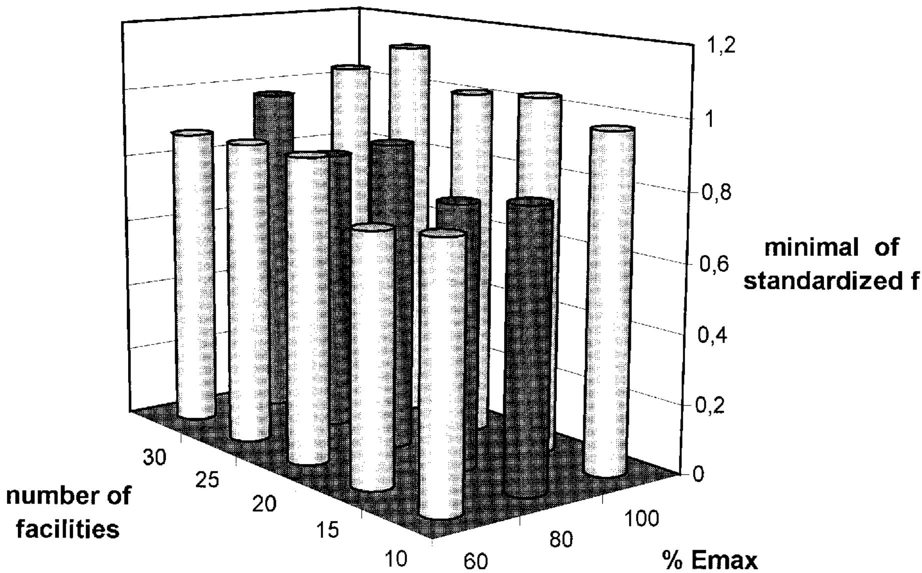

Figure 9. The 3D plot of the main relationship obtained in the second series of experiments.

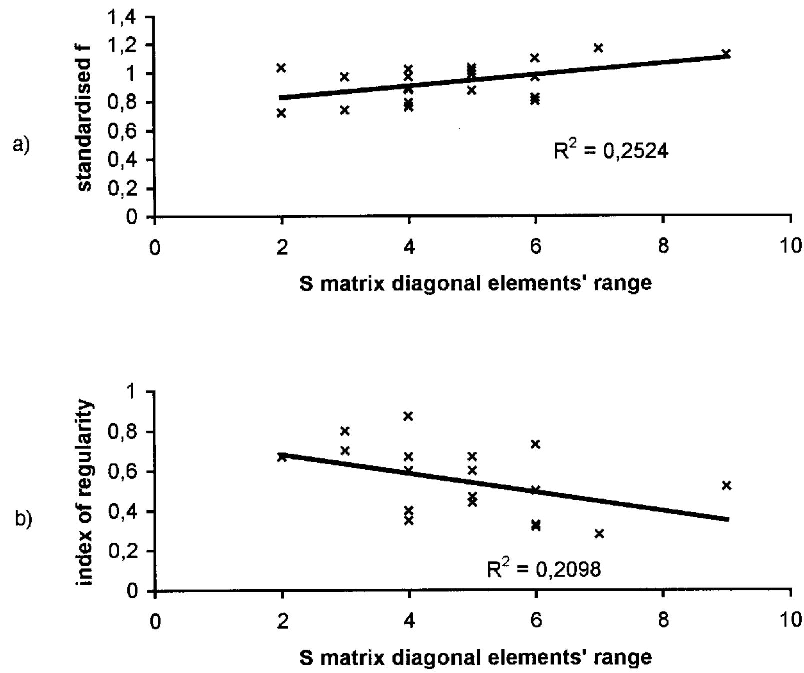

Figure 10. Relationships between the S matrix diagonal elements' ranges and the results of experiments: (*a*) standardized f, (*b*) index of regularity obtained by Drezner's method.

equal to the number of facilities N and $K = A \times A$, where A is a number of zones generating cells on X and Y. The index of regularity was defined as a number of cells containing any facility of the total number of cells (K). To make the index comparable it was standardized by multiplying it by a factor K/N. In this way the maximal

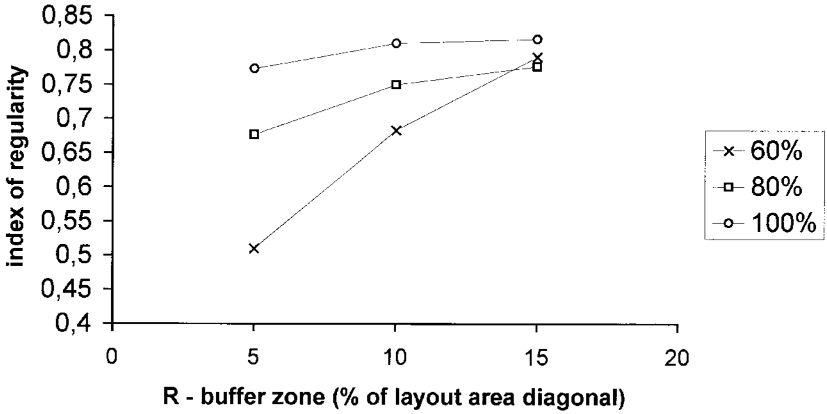

Figure 11. Index of regularity for the VF algorithm (average maximal results for different problem sizes) versus R and E_{max} percentage.

value of the index was equal to 1 when each facility is located in a separate cell. Such a defined index was calculated for all eigenvalue method results and is presented in the last column of table 4. The analysis of correlation has shown that the regularity of scatter plots depends only slightly on the regularity of links in the matrix S, measured by the variance ($r = -0.38$) or the diagonal values range ($r = -0.46$). This relationship is shown in figure 10(b). It indicates rather clearly that a possibility of the scatter plots' regularity shaping is rather limited since only one scatter plot is generated for a given S matrix, while on the other hand a matrix S is given then shaped by a designer. Performance of the VF algorithm is better in this matter. The average regularity index of plots generated by this method strongly depends on the percentage of E_{max} and the R parameter (buffer zones radius) Figure 11 illustrates this relationship for the data obtained in a special series of experiments in which the highest regularity index was taken from five trials of each of 10, 20 and 30 element problems, with 5, 10 and 15% levels of R. The analysis of variance has shown that the presented relationships are significant ($p < 0.001$ for both independent variables).

6. Summary

The results presented here give some evidence that the method of seeking scattered layouts of facilities on a plane proposed by Drezner (1987) must be applied with care. Generally, the quality of results (measured by the objective function f) depends on the data structure in this approach. Best results can be expected for problems with a great number of links. It seems that E_{max} can be a good orientation point in this matter. If the number of interconnections in a given problem is close to this level one can expect a good resolution from the eigenvector approach. Additionally, the size and structure parameters of the input matrix (S) should be taken into account. A great number of facilities being laid out and significant differences in links between facilities (diagonal elements of the matrix S) increase the probability of getting a good result.

The Virtual Force algorithm proposed in this work, as a tool of the Drezner approach evaluation, has shown its good performance in cases opposite from those described above. It gives better results especially for a smaller number of links

($E < E_{max}$). To obtain the best results one should know that, in the proposed approach, the R parameter (the buffer zones size) must be set to a value smaller than 10% of the layout region diagonal length. It is only below this limit that the VF algorithm gives solutions better then Drezner's approach. It is very likely that the general reason for the above is the observed inconsistency between assessments of f and ff functions for resolutions obtained for such small R settings in executed experiments (eigenvectors are directed on the optimization of the ff function).

The VF algorithm enables the designer to take into account sizes of facilities by setting the level of R. This factor influences regularities of generated plots very significantly. There is no such possibility in Drezner's approach.

References

Drezner, Z., 1980, DISCON a new method for the layout problem. *Operations Research*, **28**, 1375–1384.

Drezner, Z., 1987, A heuristic procedure for the layout of a large number of facilities. *Management Science*, **33**, 907–915.

Moore, J. M., 1980, The zone of compromise for evaluating lay-out arrangements. *International Journal of Production Research*, **18**, 1–10.