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On one possible 'fuzzy' approach to facilities layout problems

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A proposal of formalization of the fuzzy approach to the problem of facilities layout design is presented. An exemplary concept of fuzzy 'construction' type algorithm based on the idea of HC-66 method is introduced and illustrated by a numerical example. Possibilities of fuzzy modelling of some special cases of layout problem are discussed. The problem of theoretical predominances of a fuzzy model over the approaches similar to AEIOUX-scale is also discussed.

1. Introduction

In the review work of Karwowski and Evans (1986) many trends were presented concerning the application of the fuzzy set theory concepts in studies on production management. One of the domains distinguished by the authors concerns facilities planning, which includes such problems as facilities layout design and material handling system design. The authors notice that many variables or relationships relevant to the models existing in the above problems 'are initially specified in an imprecise and vague manner and later these are simplified for ease of analysis in an attempt to eliminate or reduce fuzziness. For example, the distance between planned facilities may be expressed as being short, medium or long. In some instances, it may be even beneficial in a design process to develop and utilize such verbal descriptors of the distance magnitude, rather than using strict values as approximations for the desired magnitudes'.

This paper presents formalization of the fuzzy approach to the problem of facilities layout design. In § 2, some basic concepts of fuzzy methodologies are given. An exemplary 'overall conception' of the fuzzy approach is presented in §§ 3 and 4. This approach is based on the classical method of Hillier and Connors (1966). The algorithm described is a fuzzy version of HC-66 algorithm. Next, we show some potential applications of the presented approach for other types of facilities layout problems and discuss the problem of a 'theoretical predominance' of the proposed concept over the Richard Muther's AEIOUX scale and other similar approaches. Some of the instances in which it may be beneficial in a design process to develop and utilize fuzzy data rather than precise values as approximations of desired magnitudes are also discussed. The last section presents conclusions and general remarks on the use of fuzzy methodologies in production management.

2. Basic terms of the theory of fuzzy sets

The notion of 'the grade of membership' constitutes the basic term introduced by Zadeh (1973). In the common theory of sets, a given object belongs or does not belong to a given set. In the two-valued logic a given term is classified as *true* or *false*. The introduction of the grade of membership makes it possible to widen both, the notion

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of belonging to a set and the principles of the classical logic. Intuitively, a fuzzy set can be understood as a class of objects in which there is no sharp boundary between objects belonging or not belonging to that class. Formally, if $X=\{x\}$ is a set of objects, then the fuzzy set A in X is said to have a membership function in which the range of the values is $[0,1]$, i.e.

$$A:X\rightarrow[0,1] \tag{1}$$

whereas

$$A=\{A(x),x\}, \qquad \text{for all } x\in X \tag{2}$$

so eventually fuzzy set A is a set of ordered pairs of the form in eqn. (2). Function $A(x)$ determines the grade of membership (belonging) of an element x in (to) the set A.

By analogy to the classical theory of sets, the notions of an intersection of fuzzy sets and a sum are introduced. Let A and B be fuzzy sets of the membership functions $A(x)$ and $B(x)$, respectively, then the sets $C=A\cap B$ and $D=A\cup B$ can be defined by means of their membership functions as follows:

$$C(x)=\min\{A(x),B(x)\}\forall x\in X \tag{3}$$

$$D(x)=\sup\{A(x),B(x)\}\forall x\in X \tag{4}$$

Fuzzy sets can provide a convenient tool to represent some simple linguistic variables concerning the levels and the intensity of certain features (Zadeh 1973, Karwowski and Evans 1986).

Operations (3) and (4) in the case of operating with sets representing linguistic variables are represented by the conjunctions 'and' and 'or'.

In the theory of fuzzy sets much attention is paid to the problems of the estimation of truth and procedures of inference (Zadeh 1978). The truth value of a given statement 'p' in respect to the criterion 'r' can be defined as 'consistency'. Let $p=$'A is F', $r=$'A is G', where A is the name of a variable, F and G denote fuzzy sets (determining the 'level of intensity' in the space X), then

$$\text{Cons } (A \text{ is } F,\ A \text{ is } G)=\text{POSS } (A \text{ is } F/A \text{ is } G)$$

$$=\sup_{x\in X}\{F(x)\wedge G(x)\} \tag{5}$$

where $\wedge$ denotes a minimum operator.

But POSS—possibility—is the category introduced by Zadeh (1978), the numerical value of which is calculated from the last element of formula (5) and the intuitive interpretation of which (in the case under discussion) lies in the examination of the 'grade of closeness' of sets F and G, or the possibility of the fact the variable A (which we know) equals G and at the same time is F (or if $A=F$ satisfies the criterion G).

The truthfulness of the implication $A\Rightarrow B$, denoted by $|A\Rightarrow B|$ (where $|A|$ means the grade of truth of the expression A, $|B|$ means the grade of truth of the expression B) is calculated from

$$|A\Rightarrow B|=\min\{1,1-|A|+|B|\} \tag{6}$$

This fact corresponds to the definition of implication in the infinite valued logic by Łukasiewicz (Zadeh 1978).

3. Aggregation of experts' assessments— a preliminary stage of solving facilities layout problems (FLP)

Gavett and Plyter (1966), when presenting the solution of the layout problems by means of the branch and bound approach, noticed that it was an extremely difficult task to obtain objective and reliable data concerning the relationships between the located objects. Since it is often impossible to define input data precisely, it can be reasonable to assume that final determination of these relationships should be the result of the aggregation of the opinions of a group of experts.

A simple version of the aggregation system based on the methodology proposed by Zadeh (1978) and adapted to the problems of 'expert-systems' by Yager (1982) is presented. For the sake of simplification it is assumed that the group of N experts has been given the task of defining the degrees of links of K objects (in pairs). It can be assumed that using the linguistic expressions such as: small, big, mean, a finite list of such expressions can be specified.

Let B_l denote the lth expression out of such a list of M possible expressions of links, $B_l(x)$—the membership function of the fuzzy set representing the expression B_l is space X. The opinion about the link value between the facilities 'i' and 'j' given by the kth expert will be denoted by L_{ij}^k. $L_{ij}^k(x)$ is the membership function of the fuzzy set representing this opinion in, for example, an artificial (not connected with any physical value) space X. The way of constructing the fuzzy sets representing linguistic expressions used in our approach and the choice of appropriate universes for these representations are essential here. Some rationale for such problems was formulated by Zadeh (1973). Attention to the relevant questions of that kind is given by Freksa (1982) and Yager (1982), Because the above problems are complex and the concepts presented here are independent of the above questions, they will not be discussed here.

For a given set of N opinions for a given pair of objects (i,j), the aggregation of these opinions can be done as follows. The choice of an appropriate expression (out of the list of expressions—B) which fits the best set of N opinions offered by individual experts constitutes the basis for such aggregation. Using formula (5), for all k, l, we can calculate

$$C_{k,l}^{(i,j)} = \mathrm{Cons}\,(L \text{ is } L_{ij}^{(k)},\ L \text{ is } B_l) = \mathrm{POSS}\,(L \text{ is } L_{ij}^{(k)}/L \text{ is } B_l)$$

$$= \sup_{x \in X} \{L_{ij}^{(k)}(x) \wedge B_l(x)\} \tag{7}$$

The evaluation $C_{k,l}^{(i,j)}$ is a number within the range $[0, 1]$. The number gets closer to 1 as the consistency of the expressions $L_{ij}^{(k)}$ and B_l gets greater. Calculating for each k

$$C_l^{(ij)} = \frac{\sum_{k=1}^{N} C_{k,l}^{(ij)}}{N} \tag{8}$$

we will obtain mean evaluation of consistency of the expression B_l to the set of opinions $\{L_{ij}^{(1)}, L_{ij}^{(2)}, \ldots, L_{ij}^{(N)}\}$. Obviously, the expression B_l for which $C_l^{(ij)}$ is maximal should be chosen as the 'most appropriate' representation of this set.

Additionally Yager (1982) postulates an assignment of the degree of importance to each expert—a number from the range [0, 1]. By denoting this number as E_k, then eqn. (8) can be rewritten as

$$C_l^{(ij)} = \frac{\sum_{k=1}^{N} E_k \wedge C_{k,l}^{(ij)}}{\sum_{k=1}^{N} E_k} \tag{9}$$

Although eqn. (9) widens the proposed procedure, from the practical viewpoint it would be difficult to answer the question of who, and based on what principles, would perform such evaluation of the importance of experts?

In the classical formulation of the FLP (e.g. in the approach suggested by Hillier and Connors (1966)), the relationships between the objects are not the only input. Therefore, each expert (or the group of experts) must also express an opinion as to the cost of locating each object at the given location and the distances between different locations. More precisely, the cost of a unit transported between two locations must be estimated. (This cost can be lineary dependent upon the distance, but not necessarily so.) It is proposed here that such value is represented by the term 'distance', because the term 'cost' will be used for the installation cost.

D_{ij}^k, G_{ij}^k denote opinions of the kth expert guarding the distance between locations i and j and the total cost of locating the facility i in the place j respectively. Using formulae (7) (9), and taking into account the changes in values $L{:}\rightarrow D{:}\rightarrow G$ in formula (7), one can obtain input data for the FLP. Such data is expressed in the form of the expressions from the list $B_1, B_2, \ldots, B_M$ represented in the given space X as appropriate fuzzy sets with the membership functions: $B_1(x)$, $B_2(x) \ldots, B_M(x)$.

4. An exemplary approach to FLP

A fuzzy modelling approach to the problem of facilities layout is the second stage of the proposed approach. The model utilizes a method of solving the layout problem in the situation when the approximate data, obtained as aggregates of experts' opinions based on the methodology described in the previous sections, are given in the form of 'fuzzy opinions'.

Since the input data have been formulated in the form of expert opinions expressed in a natural language, the construction of the appropriate criteria and the algorithmic procedure should be based upon expressions which are close to the natural language. Using such expressions, the idea of solving layout problems, as given by Hillier and Connors (1966), can be presented as the following problem. Given the cost of installation of every facility at every place, the degree of interaction intensity within every pair of facilities, and the distance between every two places, locate n facilities at n places in such a way so that the location costs and the costs resulting from interactions of facilities are minimal.

From the above, the intuitive principle of solving the FLP can be summarized as follows:

1. If the facilities i and j are *closely connected* with each other, then they should be located at the places k and r being *close to each other*.
2. The facility i should be located at the place k, for which the cost of location is *minimal*.

These statements can be treated as both the criteria of evaluating a given layout and the recommendation when solving layout problems. Also, the expressions in italics qualifying some (physical) values are formulated imprecisely. By rewriting statements 1 and 2 in a more formal way, one can obtain the criteria in the form of linguistic patterns of facilities layout.

(*a*) If the link value (of two given facilities) = "very big" then the distance (between their location places) = "very small". (10)

(*b*) Layout cost (of a given device at a given place) = "minimal". (11)

Although the above expressions of patterns may be more complicated in some of the problems, they can be interpreted using categories of fuzzy sets in appropriately defined spaces.

Let L_{ij} be a fuzzy set in the space X, determining the link value between facilities 'i' and 'j' with the membership function $L_{ij}(x)$, whereas D_{kr} be a fuzzy set in the space Y, determining the distance (transportation cost) between k and r places with the membership function $D_{kr}(y)$.

Assuming the above, it is possible to use the methodology proposed by Zadeh (1978), to calculate the truth value of L_{ij} in reference to the proposition—criterion from the expression (*a*) link value = "very big" as follows:

$$p^{(ij)} = \mathrm{POSS}(L_{ij} \text{ is v. big}) = \sup_{x \in X} (L_{ij}(x) \wedge \text{v. big}(x)) \tag{12}$$

where v. big(x) is membership function of the expression "very big" in the space X, and $\wedge$ is minimum operation. By analogy

$$q^{(kr)} = \mathrm{POSS}(D_{kr} \text{ is v. small}) = \sup_{y \in Y} (D_{kr}(y) \wedge \text{ v. small}(y)) \tag{13}$$

where v. small(y) is membership function of the expression very small in the space Y, and $q^{(kr)}$ is truth value of D_{kr} in relation to the proposition: distance = very small.

The estimation of the truth value of the implication $L_{ij} \Rightarrow D_{kr}$ (in relation to the linguistic pattern (*a*) may be performed according to the multi-modal logical formula by Lukasiewicz (Tsukamoto 1979) as

$$O_{ij}^{kr} = \min(1, 1 - p + q^{(kr)}) \tag{14}$$

where O_{ij}^{kr} can be interpreted as a degree of satisfaction for the criterion (*a*) if the facilities i and j are laid out at k and r places, respectively. Formula (14) describes the truth value of a generalized implication and, has the following properties:

$$1^\circ\ I(a, 1) = 1$$

$$2^\circ\ I(0, a) = 1$$

$$3^\circ\ I(1, a) = a$$

$$4^\circ\ I(a, b) \geqslant I(a, c) \qquad \text{if } b \geqslant c$$

$$5^\circ\ I(a, b) \geqslant I(c, b) \qquad \text{if } a \leqslant c$$

where $|a \Rightarrow b| = I(a, b)$.

A similar formulation can be proposed for the estimation of a chosen facility-location system according to the criterion (*b*).

$$Q_i^k = \mathrm{POSS}(K_{ik} \text{ is Minimal} = \sup_{c \in C} (K_{ik}(c) \wedge \mathrm{MINI}(c)) \tag{15}$$

where K_{ik} is cost of the installation of the ith facility at the kth place, $K_{ik}(c)$ and $\mathrm{MINI}(c)$ are membership functions of the fuzzy sets representing installation cost K_{ik} and 'minimal' cost in the space C, respectively. Formula (15) enables one to calculate the truth value of the criterion (*b*) being satisfied for the i facility laid out at the kth location place.

Assuming, that the number of location places (n) equals that of the laid out facilities, evaluation of a given layout system $p = p_1, \ldots, p_n$ in which p_i is the number of location to which the ith facility is assigned, in relation to the linguistic pattern (a) can be formulated as follows:

$$O(p) = \frac{1}{n^2 - n} \cdot 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} O_{ij}^{p_i p_j} \tag{16}$$

Similarly for the pattern (b):

$$Q(p) = \frac{1}{n} \sum_{i=1}^{n} Q_i^{p_i} \tag{17}$$

Estimators $O(p)$ and $Q(p)$ represent 'the mean truth value' per pair of facilities and one facility, respectively, resulting from satisfying the requirements specified by the patterns (a) and (b), in a given layout characterized by permutation p.

A general truth value (in respect of both criteria jointly) must be a result of the aggregation of O and Q estimates. In the case of operating on the estimates of truth values, the following function, with the properties given below, should be considered.

$$T: [0, 1]^2 \to [0, 1]$$

for which

1° $a < (a, 1) < 1$

2° $T(a, b) = T(b, a)$

3° $T(a, b) \geqslant T(c, d)$ if $a \geqslant c$ and $b \geqslant d$

4° $T(a, b) > T(a, d)$ if $b > d$

which corresponds to an intuition that the increase in truth of one component causes an increase in a general estimate. The mean value, which is the one that satisfies the above properties, can be written as follows:

$$G(p) = T^+(O(p), Q(p))/2 \tag{18}$$

where T^+ denotes a function satisfying the peoperties 1°–4°.

In the proposed concept, the criteria for the optimal layout of the facilities in a given space (linguistic patterns) allow for estimation of 'pure' fuzzy problems as well as the mixed ones where the criterion of distance is determined as a range (or fuzzy set) and the distance between places D_{kr} can be measured accurately.

A particular case of the proposed approach is the formulation of estimates and criteria as intervals. For example, criterion (a) can be represented as

$$\text{If link value} = \text{'A'} \quad \text{Then the distance} = \text{'B'} \tag{19}$$

where 'A' and 'B' denote 'sharply' determined numerical intervals in corresponding spaces, then

$$p = \mathrm{POSS}(L_{ij} \text{ is } A) = \sup_{x \in X} (L_{ij}(x) \wedge A(x))$$

$$\begin{cases} = 0 & \text{if } L_{ij} \cap A = \phi \\ = 1 & \text{if } L_{ij} \cap A \neq \phi \end{cases}$$

and

$$q^{(kr)} = \mathrm{POSS}(D_{kr} \text{ is } B) = \sup_{y \in Y} (D_{kr}(y) \wedge B(y))$$

$$\begin{cases} = 0 & \text{if } D_{kr} \cap B = \phi \\ = 1 & \text{if } D_{kr} \cap B \neq \phi \end{cases}$$

$L_{ij}(x)$ and $A(x)$, as well as $D_{kr}(y)$ and $B(y)$, are fuzzy sets with membership function equal to 1 for a given interval and 0 beyond it. The truth value of satisfying the pattern (criterion (19)) by a given layout system is now calculated according to the classical implication formula.

The proposed model enables the evaluation of any facilities layout system and makes it possible to choose the best one. The above problem can be formulated as that of discrete optimization: max $G(p)$, where p is the set of all permutations of numbers $1, 2, \ldots, n$. The review of all possible variants of the layout for $n > 10$ is ineffective even for a fast computer. A precise branch and bound method (for classical FLP problems) based on the solution of the travelling salesman problem, is effective only for $n < 10$ (Gavett and Plyter (1966)). Foulds (1983), presenting the results of the work by Sahni and Gonzales, stated that an effective algorithm for the problem with dimension $n > 10$ is unlikely to be found since it is an NP-complete problem. Hence, the attempts to find approximate methods are of great importance. Many of these heuristic methods have been discussed by Foulds (1983) and by Bonney and Williams (1977). In §5, the heuristic algorithm, based on the HC-66 algorithm concept (Hillier and Connors 1966), is presented.

5. Heuristic algorithm for the exemplary approach

Assuming that in the case of (a)-(b) patterns being applied as criteria, the following parameters are given (in the form of fuzzy sets):

1. the interrelationship degrees for facilities (link values)—matrix $L = |L_{ij}|_{nxn}$,
2. distances between location places—matrix $D = |D_{kr}|_{nxn}$, and
3. costs of installation of each facility in each possible place—matrix $K|K_{ij}|_{nxn}$.

If the number of facilities n equals that of possible places (which does not change the generality degree of considerations) one should follow the following algorithm:

1° Create the matrix L', where an element $L'_{ij} = p^{(i,j)}$ (formula (12)) and the matrix D', where the element $D'_{kr} = q^{(kr)}$ (formula (13)) for $i = 1, \ldots, n$, $k = 1, \ldots, n$, j

$=1, \ldots, n$, $r=1, \ldots, n$, assuming for all $i=j$ and $k=r$, $L'_{ij}=D'_{kr}=1$. Substitute $RL' := L'$ and $RD' := D'$.

2° Create matrix F of elements $F_{ik}=Q_i^k$ (formula (15)) for all 'i' and 'k'.

3° Sort out the elements in columns of matrices RL' and RD' in a decreasing order, denoting the obtained matrices by L'' and D'', respectively.

4° Determine the matrix of assignments—A ('truth losses') according to the following 'step criterion functions':

$$A_{kl}=1-\tfrac{1}{2}\left(F_{kl}+\frac{a+b}{2}\right) \tag{20}$$

where

$$a=\frac{1}{|\mathscr{L}|}\sum_{i\in\mathscr{L}}((1-L'_{ki}+D'_{lpi})\wedge 1) \tag{21}$$

and

$$b=\frac{1}{n-|\mathscr{L}|}\sum_{j=1}^{n-|\mathscr{L}|}((1-L''_{jk}+D''_{jl})\wedge 1) \tag{22}$$

in which $|\mathscr{L}|$ is cardinality of set of laid out elements; $\mathscr{L}$ is set of indices of layed out elements, and pi is index of the location place of 'i' facility. (For the 'first pass' $|\mathscr{L}|=0$ and $\mathscr{L}=\phi$, a is assumed to be 0.)

5° Choose an assignment of the kth facility to the lth place so that it will provide the minimum 'truth loss', applying the VAM method (Vogel's approximation method, (see Appendix 1); substitute $\mathscr{L} := \mathscr{L}-\{k\}$.

6° Delete the rows and the columns in the matrices RL', RD' corresponding to the chosen facility and its location place.

7° If one facility has been left, locate it at a remaining place, otherwise repeat the algorithm once more, beginning with step point 3°.

In order to illustrate this algorithm a simple numerical example has been presented in §6.

It should be noticed that the proposed algorithm is based on the branch and bound method. The basic difference between them is lack of 'returns' to once rejected branches of the 'solution tree'. Also, only one part of 'step criterion function' (eqn. (20)) is estimated by finding its upper bound (eqn. (21)). The matrix A_{kl}, based on which assignments in the proposed algorithm are performed, may be interpreted as a matrix of 'truth value losses', associated with the satisfaction of the (a) and (b) criteria selecting a given layout. The lowest loss of the truth value is gained for the layout providing the maximum value for the expression in brackets in eqn. (20). The part denoted by a in eqn. (21) enables one to determine the potential for increase of the truth value per each new link of already laid out facilities, resulting from the location of the 'k' facility at the 'l' place (or the mean truth value of the k facility located at the l place, satisfying criterion (a) together with the facilities having already been laid out).

'b' (in formula (22)) is an estimate of the upper bound of the truth value for the prospective increase of the (a) criterion satisfaction, resulting from the location of the 'k' facility at the lth place. One can formally rewrite

$$b=\max_{p\in\pi}\frac{1}{n-|\mathscr{L}|}\sum_{j=1}^{n-|\mathscr{L}|}((1-RL'_{kp_j}+RD'_{lj})\wedge 1) \tag{23}$$

where π is set of all permutations of indices of facilities not laid out, p is permutation of indices of facilities not laid out, and p_j is the jth element of permutation p.

It is possible to rewrite the formula (23) in the form b of the formula (22) due to the application of 'the maximum truth value theorem' presented together with its proof in Grobelny (1985). See Appendix 2.

6. Simple example problem

Let us assume that four experts are involved in a problem of estimating data for a given plant layout. They formulate opinions on link values of facilities, distance values between places and installation costs. Each expert uses five linguistic expressions which are fuzzy sets defined on appropriate universes. Definitions of the expressions are given in Tables 1–3.

Let us assume that for L_{13} – link value, the human experts gives the following estimates: $L_{12}^{(1)}=$ v. big, $L_{12}^{(2)}=$ v. small, $L_{13}^{(3)}=$ medium, $L_{14}^{(4)}=$ v. big. For the sake of

	X (artificial universe)				
	1	2	3	4	5
1. Very small (x)	1·0	0·2	0·0	0·0	0·0
2. Small (x)	0·9	1·0	0·5	0·1	0·0
3. Medium (x)	0·1	0·8	1·0	0·8	0·1
4. Big (x)	0·0	0·1	0·5	1·0	0·9
5. Very big (x)	0·0	0·0	0·0	0·2	1·0

Table 1. Representation of the linguistic expressions as fuzzy categories in the universe X.

	Y (distance universe certain length units)				
	10	20	30	40	50
1. Very small (y)	1·0	0·2	0·0	0·0	0·0
2. Small (y)	0·9	1·0	0·5	0·1	0·0
3. Medium (y)	0·2	0·8	1·0	0·8	0·1
4. Big (y)	0·0	0·1	0·5	1·0	0·9
5. Very big (y)	0·0	0·0	0·0	0·2	1·0

Table 2. Representation of the linguistic expressions as fuzzy sets in the distance (cost) space Y.

	C (cost universe in $)				
	10000	20000	30000	40000	50000
1. Very small (c)	1·0	0·2	0·0	0·0	0·0
2. Small (c)	0·9	1·0	0·5	0·1	0·0
3. Medium (c)	0·1	0·8	1·0	0·8	0·1
4. Big (c)	0·0	0·1	0·5	1·0	0·9
5. Very big (c)	0·0	0·0	0·0	0·2	1·0

Table 3. Representation of the linguistic expressions as fuzzy sets in the space C.

simplification it is assumed that the set of 'possible expressions of links'—B (§3) contains 5 expressions defined in Table 1.

Using formula (7) one can calculate:

$$C_{1,1}^{(1,2)} = \text{POSS}(L \text{ is } L_{12}^{(1)}/L \text{ is } B_1) = \sup_{x \in X} (\text{v. big}(x) \wedge \text{v. small}(x))$$

$$= \sup \{0{\cdot}0, 0{\cdot}0, 0{\cdot}0, 0{\cdot}0, 0{\cdot}0\} = 0{\cdot}0$$

By analogy:

$$C_{2,1}^{(1,2)} = 1{\cdot}0,\ C_{3,1}^{(1,2)} = 0,2,\ C_{4,1}^{(1,2)} = 0{\cdot}0$$

Now using eqn. (8)

$$C_1^{(1,2)} = \frac{1,2}{4} = 0,3$$

The interpretation of the value $C_1^{(1,2)}$ is that the 1st expression from Table 1 (very small) is consistent with the set of experts' opinions on 1–2 link value with the grade 0·3. One can also check that the best for representation of experts opinions as to the 1–2 link value level is 'big' since

$$\max_i (C_i^{(1,2)}) = C_4^{1,2} = 0{\cdot}67.$$

Matrix L, is shown below, the result of calculations similar to those presented above.

$$L = \begin{array}{c} \text{Facility number} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \\ \left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline x & \text{Big} & \text{Medium} & \text{V. small} \\ & x & \text{Big} & \text{V. small} \\ & & x & \text{Small} \end{array}\right) \end{array}$$

In the same way, the matrix of distances (costs of transportation) between locations can be developed. Matrix D, shown below, contains both fuzzy (linguistic) values as well as numerical estimates. This indicates flexibility of the proposed model in the face of different types of available data.

$$D = \begin{array}{c} \text{Location number} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \\ \left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline x & \text{Medium} & \text{Small} & 38 \\ & x & 15 & \text{Big} \\ & & x & 50 \end{array}\right) \end{array}$$

Table 4 contains installation cost values. Some of these values are fuzzy, others are accurately measured numbers or intervals of real numbers.

To use the proposed algorithm one should prepare given data in an appropriate form. In accordance with step 1 of the algorithm matrices L' and D' should be constructed. For example:

$$L'_{12} = p^{(12)} = \text{POSS}(L_{12} \text{ is V. big}) = \sup_{x \in X} (\text{Big}(x) \wedge \text{V. big}(x))$$

$$= \sup \{0{\cdot}0, 0{\cdot}0, 0{\cdot}0, 0{\cdot}2, 0{\cdot}9\} = 0{\cdot}9$$

Facility number	Location number			
	1	2	3	4
1	Very small	More than 35000	Very big	Very big
2	40000	More than 35000	15000	15000
3	30000–50000	Medium	25000	32000
4	10000–20000	Very big	45000	Less than 20000

Table 4. Cost matrix K (assessed by experts).

Computing in the above way (and adding 1 for L'_{ii} and D'_{ii}) we obtain

$$L' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 0{\cdot}9 & 0{\cdot}2 & 0{\cdot}0 \\ 0{\cdot}9 & 1{\cdot}0 & 0{\cdot}9 & 0{\cdot}0 \\ 0{\cdot}2 & 0{\cdot}9 & 1{\cdot}0 & 0{\cdot}1 \\ 0{\cdot}4 & 0{\cdot}0 & 0{\cdot}1 & 1{\cdot}0 \end{pmatrix} \end{array} := RL'$$

In order to constitute matrix D' an assumption is made that the membership functions of the expressions given in Table 2 are linear between discrete points of the universe Y. For example

$$D'_{23} = q^{(23)} = \mathrm{POSS}(D_{23} \text{ is V. small}) = \sup_{y \in Y}(15(y) \wedge \text{V. small}(y))$$

$$= \text{V. small}(15) = -0{\cdot}08 \cdot 15 + 1{\cdot}8 = 0{\cdot}6$$

where $15(y)$ means singleton, i.e. fuzzy set with only one element (15), and membership grade equal to 1.

Then the matrix D' is as follows:

$$D' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 0{\cdot}2 & 0{\cdot}9 & 0{\cdot}0 \\ 0{\cdot}2 & 1{\cdot}0 & 0{\cdot}6 & 0{\cdot}1 \\ 0{\cdot}9 & 0{\cdot}6 & 1{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}1 & 0{\cdot}0 & 1{\cdot}0 \end{pmatrix} \end{array} \quad (:= RD')$$

Assuming that expressions more than 'a', less than 'b' are 'normal' (non-fuzzy) sets defined in the cost universe†, $a-b$, 'normal' interval and 'a' normal number (singleton) in C universe, one can construct matrix F as follows (step 2 of the procedure):

$$F_{12} = \mathrm{POSS}(\text{'More than 35000' is V. small}).$$

$$= \sup_{c \in C}(>35000(c) \wedge \text{V. small}(c)) = 0{\cdot}0$$

† In the 'fuzzy language' it means that all elements which belong to these sets have a membership grade equal to 1.

where more than 35000 (c) is the fuzzy definition of normal set of numbers >35000 in universal C (i.e. all numbers greater than 35000 have got the membership grade equal to 1 and the other equal to 0).

Matrix F is

$$F = \begin{array}{c|c} \text{Location facility} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}6 & 0{\cdot}6 \\ 0{\cdot}0 & 0{\cdot}2 & 0{\cdot}1 & 0{\cdot}0 \\ 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 \end{pmatrix} \end{array}$$

Sorting out elements of columns of L' and D' in descending order (step 3 of the algorithm) one obtains

$$L''^{(0)} = \begin{pmatrix} 1{\cdot}0 & 1{\cdot}0 & 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}9 & 0{\cdot}9 & 0{\cdot}9 & 0{\cdot}1 \\ 0{\cdot}4 & 0{\cdot}9 & 0{\cdot}2 & 0{\cdot}0 \\ 0{\cdot}2 & 0{\cdot}0 & 0{\cdot}1 & 0{\cdot}0 \end{pmatrix} \qquad D''^{(0)} = \begin{pmatrix} 1{\cdot}0 & 1{\cdot}0 & 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}9 & 0{\cdot}6 & 0{\cdot}9 & 0{\cdot}1 \\ 0{\cdot}2 & 0{\cdot}2 & 0{\cdot}6 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}1 & 0{\cdot}0 & 0{\cdot}0 \end{pmatrix}$$

(The indices (0) means the 'first pass' of the procedure.)

In order to apply step 4 of the algorithm, we determine matrix $A^{(0)}$ for the 'first pass' according to formulae (20)–(22). For example:

$$A_{22}^{(0)} = 1 - \tfrac{1}{2}(0{\cdot}0 + \tfrac{1}{2}(0{\cdot}0 + \tfrac{1}{4}(1 + 0{\cdot}7 + 0{\cdot}3 + 1))) = 0{\cdot}81$$

The full $A^{(0)}$ matrix has the form:

$$A^{(0)} = \begin{array}{c|c|c} \text{Facilities numbers} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} & \text{Locations numbers} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 0{\cdot}27 & 0{\cdot}79 & 0{\cdot}75 & 0{\cdot}84 \\ 0{\cdot}80 & 0{\cdot}81 & 0{\cdot}49 & 0{\cdot}66 \\ 0{\cdot}76 & 0{\cdot}67 & 0{\cdot}71 & 0{\cdot}82 \\ 0{\cdot}25 & 0{\cdot}75 & 0{\cdot}75 & 0{\cdot}25 \end{pmatrix} & \begin{array}{c} 0{\cdot}48 \\ 0{\cdot}17 \\ 0{\cdot}04 \\ 0{\cdot}0 \end{array} \\ & \begin{array}{cccc} 0{\cdot}02 & 0{\cdot}08 & 0{\cdot}22 & 0{\cdot}41 \end{array} & \text{The 'VAM' indices} \end{array}$$

According to the 'VAM' method (see Appendix) we can choose the first location. We assign facility numbered 1 to the location which is 1. In this way, the 5th step of the proposed procedure is completed. Realization of the step 6 of the algorithm gives

$$RL' = \begin{array}{c|c} & \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 1{\cdot}0 & 0{\cdot}9 & 0{\cdot}0 \\ 0{\cdot}9 & 1{\cdot}0 & 0{\cdot}1 \\ 0{\cdot}0 & 0{\cdot}1 & 1{\cdot}0 \end{pmatrix} \end{array} \qquad RD' = \begin{array}{c} \begin{array}{cccc} & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 2 & 1{\cdot}0 & 0{\cdot}6 & 0{\cdot}1 \\ 3 & 0{\cdot}6 & 1{\cdot}0 & 0{\cdot}0 \\ 4 & 0{\cdot}1 & 0{\cdot}0 & 1{\cdot}0 \end{pmatrix} \end{array}$$

Now, because there is more than one non-assigned facility (step 7) we go to step 3 and sort out the elements in columns of RL' and RD'. We obtain

$$L''^{(1)} = \begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}9 & 0{\cdot}9 & 0{\cdot}1 \\ 0{\cdot}0 & 0{\cdot}1 & 0{\cdot}0 \end{pmatrix} \end{array} \qquad D''^{(1)} = \begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}6 & 0{\cdot}6 & 0{\cdot}1 \\ 0{\cdot}1 & 0{\cdot}0 & 0{\cdot}0 \end{pmatrix} \end{array}$$

The new 'true losses' matrix $A^{(1)}$ will be constructed according to the formulae (20)–(22) as in the following example:

$$A^{(1)}_{22} = 1 - \tfrac{1}{2}[0{\cdot}0 + \tfrac{1}{2}(0{\cdot}3 + \tfrac{1}{3}(1 + 0{\cdot}7 + 1))] = 0{\cdot}7$$

The final $A^{(1)}$ matrix is as follows:

$$A^{(1)} = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0{\cdot}7 & 0{\cdot}23 & 0{\cdot}45 \\ 0{\cdot}43 & 0{\cdot}52 & 0{\cdot}62 \\ 0{\cdot}55 & 0{\cdot}5 & 0{\cdot}10 \end{pmatrix} \\ \begin{array}{ccc} 0{\cdot}12 & 0{\cdot}27 & 0{\cdot}35 \end{array} \end{array} \begin{array}{c} \\ 0{\cdot}22 \\ 0{\cdot}09 \\ 0{\cdot}40 \\ \\ \end{array}$$

The chosen assignment is 4→4 (facility of 4 numbers in the 4th place). Matrices RL', RD', $L''^{(2)}$, $D''^{(2)}$ for the third pass are as follows:

$$RL' = \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 0{\cdot}9 \\ 0{\cdot}9 & 1{\cdot}0 \end{pmatrix} \end{array} \qquad RD' = \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 0{\cdot}6 \\ 0{\cdot}6 & 1{\cdot}0 \end{pmatrix} \end{array}$$

$$L''^{(2)} = \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}9 & 0{\cdot}9 \end{pmatrix} \end{array} \qquad D''^{(2)} = \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{pmatrix} 1{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}6 & 0{\cdot}6 \end{pmatrix} \end{array}$$

And the final $A^{(2)}$ matrix in the form

$$A^{(2)} = \begin{array}{c} \text{Facility} \\ \text{number} \\ 2 \\ 3 \\ \\ \end{array} \begin{array}{c} \\ \begin{array}{cc} 2 & 3 \end{array} \\ \begin{pmatrix} 0{\cdot}73 & 0{\cdot}24 \\ 0{\cdot}44 & 0{\cdot}5 \end{pmatrix} \\ \begin{array}{cc} 0{\cdot}29 & 0{\cdot}26 \end{array} \end{array} \begin{array}{c} \text{Location} \\ \text{number} \\ 0{\cdot}49 \\ 0{\cdot}06 \\ \\ \end{array}$$

We choose 2→3 location and the remaining facility (i.e. 3) is assigned to the remaining place (i.e. 2). It is the end of procedure. The diagram of layout obtained is shown in Figure 1.

Analysing cost matrix C' we can see that a given plant layout is the best one if only location cost is taken into account. Applying the 'maximal mean truth value' theorem (see Appendix 2) to the data given in matrices L, D, L', D', one can find the upper bound on the values of $\theta(p)$. In this example, the upper bound is equal to 0·95. Therefore, the solution given by the proposed algorithm is optimal (comparing it with value of $\theta(p)$ for a layout given by the algorithm).

7. Possible developments of linguistic patterns

It should be noted that the proposed algorithm is of the 'construction' type and is one of many possible solutions to the problem presented. It does, however, show that the idea of branch and bound may be used in the problems of layout planning with fuzzy data. Grobelny (1986) proposed the application of this idea shown for a search of optimal solutions according to the classic approach of Gavett and Plyter (1966). However, having a set of linguistic patterns, one can directly use some of the iterative methods, for example CRAFT.

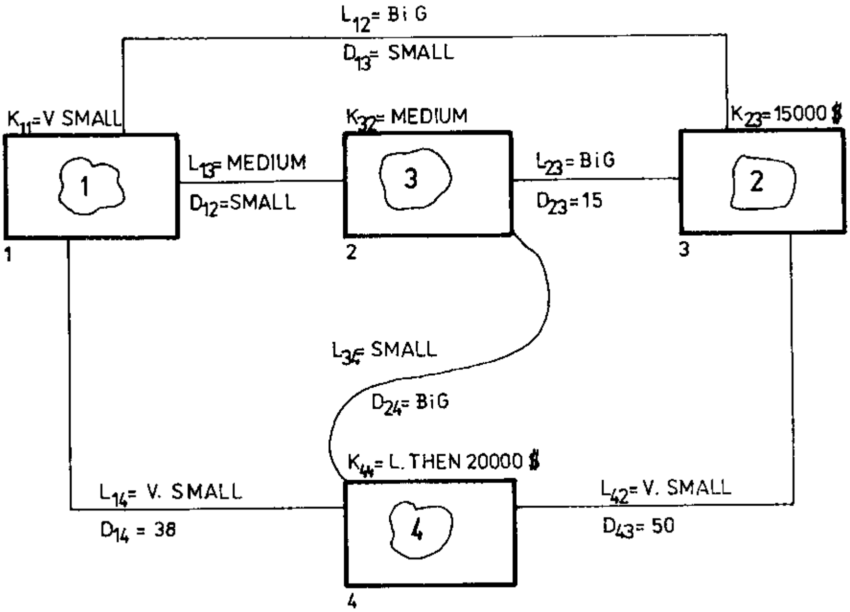

Figure 1. The diagram of a layout given by proposed fuzzy algorithm.

The use of linguistic patterns expressed in the form IF a THEN b does not impose too many constraints on the forms of expressions a and b. Rather, it enables one to construct specific sets of criteria (patterns), adapted to an actual situation, and to take into account a lot of other variables. For example, using a fuzzy relation as a representation of an expression 'A and B'† (Zadeh 1973, Karwowski and Evans 1986), the following example of linguistic pattern can be introduced for every pair of facilities in the given layout.

If the link value = 'very big' and the cooperation
time = 'very long' then the distance = 'very small'

Thus one can obtain a simple model of a certain dynamic problem in which the estimation of arrangement of facilities depends on the intensity of interrelationships (link values) as well as on the expected length of time (cooperation time) of these interrelationships. Assuming that an approximate working time of a given system of facilities is known that one can estimate (in a fuzzy manner) the cooperation times of particular pairs of facilities (starting from the end), then the above linguistic patterns may be a basis to formulate an arrangement including the expected development (i.e. an inclusion of new devices and or the new links in time).

† I.e. if A is a fuzzy subset in a universe of discourse U, and B is a fuzzy subset in a universe of discourse V, then the cartesian product $A \times B$ is defined as a fuzzy relation:

$$A \times B = \sum_{R = U \times V} (A(u) \wedge B(v))/u, v)$$

where $\sum$ is a symbol of union, $\wedge$ is a minimum operator, $C(u)/u$ denotes an element u of the membership function $C(u)$. R is usually given in the form of a matrix.

Eastman (1973) noticed that the problem of an arrangement is a very complicated and multi-critical process. In his approach, based on the heuristic search (automated space planning), he introduced many criteria in the form of relations which should be satisfied in any acceptable arrangement. For example, the following relations: *adjacent*, *sight*, *distance*, *orient* which denote adjacency of a given facilities pair, mutual visibility of a given pair of elements (facilities), a condition imposed on the mutual distance and a condition imposed on the orientation of parts of a given pair of devices in relation to one another, respectively, should be satisfied. Although it is obvious that these are the non-sharp relations, Eastman represents these relations in the form of classic, strict, mathematical conditions. Each of them may be introduced to the set of linguistic patterns which form a fuzzy model. The form of such patterns, determined by a definite situation, can vary. One may imagine, for example, the following criteria:

1. IF adjacency of (i, j) pair is *necessary* THEN facilities i and j are *adjacent*.
2. IF visual communication between i, j facilities is *necessary* THEN a site i from j and j from i is *possible*.
3. IF a degree of mutual work disturbance for the i–j pair is *big* THEN a distance between i and j is *long*.

When the italic expressions are defined in the form of fuzzy sets in proper universes of discourse, then for any given layout one may count a truth value of fulfilment of the above patterns by every pair of facilities, using formulae (5) and (6). However applying the approach analogical to the presented one, one may count a mean truth value for the whole arrangement (formula (16)), and next aggregage the truth values for all criteria as an arithmetic mean of a function having the 1°–4° (§4) properties. In the case of the above pattern set, one cannot directly apply the exemplary approach. In order to do so, it would be necessary to use the adequate modular net which divides the whole accessible space into individual places.

The presented examples are not the only potential possibilities of the criteria construction in the form of linguistic patterns. However, they are indicative of great possibilities in this field, especially perspectives for constructing the flexible methods of computer-aided facilities layout design.

8. The problem of 'predominance'

Any new approach, or algorithm in the field of industrial engineering induces the following basic question: what are the predominances of a new approach over the methods existing in the given field? The answer to this question consists of supplying documentary evidence for better effects of the new algorithm over the existing ones. These better effects mean that the new algorithm should give better solutions or reduce the cost of searching for solutions.

Unfortunately, such a settlement is not possible in our case. Since the proposed approach is predestined for a situation in which an arrangement is looked for in conditions of non-sharp (fuzzy) knowledge about the input data, it makes no sense to compare it with any of the classical approaches—predestined to well-defined, numerical input data.

However, one could treat the classical Muther's scale as an existing model of fuzzy data representation. The fuzziness in this model is eliminated by use of precise numbers (1–6). To prove the predominance of the fuzzy approach over Muther's-like propositions, one should use both methods in a variety of practical situations and compare the results. Some of the theoretical premises, testifying to an advantage of the fuzzy approach in a fuzzy environment, are as follows:

1. The fuzzy approach is more general than numerical representation for linguistic variables (scales of the AEIOUX type). In such an approach, one may even use exact measurements (if they are attainable) of input variables and their different levels will be differentiated. It is also possible (see 'exemplary approach') to use 'the sharp' intervals of variables which (in the range of information quality) correspond with the rank approach of Muther. Fuzzy models allows one to use less and/or more precise (fuzzy) information (expressed in the form of membership function) which is not possible in the alternative approach discussed. Such 'qualitatively differentiated' information is distinguishable only by fuzzy models and it makes them more applicable in the face of fuzzy knowledge.
2. The use of categories of logical truth gives additional information about the distance of a given solution from the 'fully true' solution (i.e. from a certain 'absolute'). In the solutions based on numerical representation, only relative comparison of two arrangement is possible. In some cases, however, it is possible to estimate the upper bounds or lower bounds as well as to apply the fuzzy approach (see maximal truth theorem, Appendix 2).
3. A general form of a linguistic pattern allows for flexible formulation of the model's criteria in a way close to the linguistic description of a situation. This may be of particular importance while constructing the flexible and interactive computer models.

There are also a few premises testifying to the approach of Muther's type. Here are the most important:

1. The fuzzy approach needs the determination of fuzzy representation of the expert's knowledge. In spite of practical research in this field (for example Freksa (1982)) commonly accepted standards (patterns) of collection of the required data and construction of a fuzzy representation of the linguistic variables do not exist. In the approach of the 'AEIOUX' type, the problem of representation is omitted by the arbitrary assignment of numbers to the particular categories.
2. At present, the classical computer algorithms utilizing the numerical approaches are more efficient than fuzzy modifications since they compute less information.

Although the definitive solution of the dominance problem requires intensive and multidirectional studies, the practical applications may be the final arbiter in this respect.

The advantages and disadvantages presented above concern not only the proposed approach, but any other foreseeable fuzzy formulation of the facilities layout problem. In every case, the essence of fuzzy approach will be fuzzy representation of the input data and the application of some instrument from the fuzzy set theory for the proper treatment of data to obtain the arrangement which best satisfies the fuzzy requirements. Therefore, in every case there will be indications of predominance of the first (1) type.

9. Concluding remarks

The presented fuzzy approach to the problem of facilities layout planning proves that the use of inaccurate data need not be connected with reduction of fuzziness—which is natural in such cases. This proposal corresponds to the opinions presented in the work of Karwowski and Evans (1986). According to these views there are such situations in which it is more convenient to use fuzzy variables. In general, it is more convenient to use inaccurate, linguistic terms rather than precise, numerical values—because the 'cognitive distance' between natural principles of the brain-work and non-sharp notions is (probably) smaller than in the case of numerical quantities (Freska 1982). Furthermore, the mistakes made during the estimation of physical quantities are the least when this estimation is based on fuzzy categories.

The opinions presented by Karwowski and Evans (1986), and also the results of Freska's studies lead to the formulation of a general thought about the relations between the precise, analytical approach and the fuzzy proposals in industrial engineering. Such thought may be expressed as 'the principle of interdetermination'. The more precise quantities are used in the model, the greater the possibility of making mistakes, although the settlements of more precise models are (apparently) more accurate. In such a situation the fuzzy approach may be one of the ways for the search of rational compromise between the precision of models and reliability of the obtained results.

Appendix 1

The sequence of calculations in the VAM method (Lis and Santarek 1980):

(*a*) The differences between the two smallest elements are calculated for each row and each column of the matrix:

$$R_i = A_{i,p_2} - A_{i,p_1}; \qquad i = 1, 2, \ldots, n$$

where $A_{i,p_2} \geqslant A_{i,p_1}$ and these are the two smallest elements in the row i.

$$S_p = A_{i2,p} - A_{i1,p}, \qquad p = 1, 2, \ldots, n$$

where $A_{i2,p} \geqslant A_{i1,p}$ are the two smallest elements in the column p.

(*b*) A row (or column) is chosen to which the greatest calculated differences correspond; it is the i_k row when

$$R_{i_k} = \max_{i,p=1,\ldots,n} (R_i, C_p)$$

or column p_k when

$$C_{p_k} = \max_{i,p=1,\ldots,n} (R_i, C_p)$$

(*c*) In the chosen row i_k (or the column p_k) of matrix A, the minimum element is sought. This is an element from the p_k column when

$$A_{i_k}, p_k = \min_{p=1,\ldots,n} (A_{i_k}. p)$$

or an element of the i_k row when

$$A_{i_k}, p_k = \min_{i=1,\ldots,n} (A_{i;p_k})$$

The facility corresponding to the i_k row is located at the p_k place. At each step matrix A lessens, and consists of different elements—thus for a given matrix A, the calculation cycle finishes after step (c) has been performed.

Appendix 2. Theorem 'of the maximum mean truth'

Let $p_1, p_2, \ldots, p_n$ denote the set of 'truth values' of the statements $A_1, A_2, \ldots, A_n$, and $p_1 \geqslant p_2 \geqslant \ldots \geqslant p_n$.

Let $q_1, q_2, \ldots, q_n$ denote the set of 'truth values, of the statements $B_1, B_2, \ldots, B_n$ and $q_1 \geqslant q_2 \geqslant \ldots \geqslant q_n$ with $p_i, q_j \in [0, 1]$.

Let $A_i \Rightarrow B_j$ denote a generalized implication and $t_{ij} = \min\ (1, 1 - p_i + q_j)$—the truth value of this implication (Tsukamoto 1983). Then the following theorem is true:

$$\max_P \left(\frac{1}{n} \sum_{i=1}^{n} t_{i\pi_i} \right) = \frac{1}{n} \sum_{i=1}^{n} t_{ii}.$$

P-set of all permutations of numbers $1, 2, \ldots, n$ and π_i—ith element of the permutation π. That means that the greatest 'mean truth value' in the set of implications $\{A_i \Rightarrow B_i\}_{i=1}^{n}$ is obtained when the statements of A and B types are ordered respectively in a non-increasing way, according to the estimates of the truth values: p_i and q_i, i.e. $A_1 \Rightarrow B_1, A_2 \Rightarrow B_2, \ldots, A_n \Rightarrow B_n$.

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Il s'agit d'une présentation de formalisation de l'approche floue au problème de la conception de disposition des facilités. Un exemple de concept de 'construction' floue de type algorithme fondé sur l'idée de la méthode HC-66 est introduit et illustré par un exemple numérique. Les possibilités de modelage flou de quelques cas particuliers de problèmes de disposition sont discutés. Le problème des prédominances théoriques d'un modèle flou part rapport aux approches similaires à l'échelle AEIOUX fait également l'objet de discussions.

Es wird ein Vorschlag zur Formalisierung des auf der 'unscharfen' Logik basierenden mengentheoretischen Ansatzes zur Lösung des Anlagen-Lageplanproblems vorgestellt. Als Beispiel wird der Begriff eines unscharfen 'konstruktionsartigen' Algorithmus eingeführt, der auf den Ideen des HC-66-Verfahrens beruht, und der an einem zahlenmäßigen Beispiel veranschaulicht wird. Es wird die Möglichkeit besprochen, gewisse Sonderfälle von Lageplanproblemen mit unscharfen Modellen nachzubilden. Ferner wird das Problem der theoretischen Überlegenheit des unscharfen Modells über andere Lösungsansätze wie die AEIOUX-Skale besprochen.